ANALYSIS OF THE FEASIBILITY OF DESCRIBING UNSTEADY HEAT CONDUCTION PROCESSES IN DENSE DISPERSE SYSTEMS BY DIFFERENTIAL EQUATIONS ACCOUNTING FOR PHASE INTERACTION CHARACTERISTICS

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The solution of a set of differential equations for unsteady heat conduction of a disperse medium is analyzed and a comparison is presented with experimental data from several authors.

The set of differential equations for unsteady heat conduction in a disperse medium [1] is based on a model representation which calls for imaging of the most essential portion of the process which is extremely difficult to describe in a rigorous mathematical manner. In selection of the model it is not possible to obtain an a priori quantitative estimate and, at times, even a qualitative estimate of the validity of simplifying assumptions. Intuitive physical considerations which form the basis for model selection, and are written in the form of differential equations, must be verified by analysis of the solution and comparison with experimental data. Often, however, even comparison of experimental and theoretical data is limited. Here it is necessary to note that agreement between theoretical and experimental values in a given case may be fortuitous and hence, the model may only be considered for interpolation or computation. A model based on qualitative approximate estimates of the complex real processes may be considered as having physical basis if the relationships obtained not only satisfy the experimental data over a wide range of variables but also do not lose their physical significance when extrapolated beyond the limits encompassed by experiment.

1. Determination of Computational Relationships. Accounting for the fact that, for the disperse systems under consideration (disperse solid phase – gas), the thermal conductivity of the solid phase is 4 to 5 orders of magnitude lower than that of the system as a whole, we simplify the initial set of differential equations for unsteady heat conduction in a disperse medium and represent them in the form

$$c_1 \rho_1 \frac{\partial \vartheta_1}{\partial \tau} = \alpha^* S (\vartheta_2 - \vartheta_1), \tag{1}$$

$$c_2 \rho_2 \frac{\partial \vartheta_2}{\partial \tau} = \lambda_2 \frac{\partial^2 \vartheta_2}{\partial x^2} - \alpha^* S (\vartheta_2 - \vartheta_1). \tag{2}$$

This simplified set of equations corresponds to a model of unsteady heat conduction in a disperse medium in which, in contrast to the initial one, [1, 8], one of the phases (namely the solid phase) is considered only as a thermal sink while the second (gas) phase determines the molecular transport of internal energy within the system.

From sufficiently general physical considerations it follows that, for disperse systems of the type, disperse solid phase–gas , the simplified model apparently corresponds closely to the physical nature of the process. We note also that these considerations are confirmed by some of the results of this work.

To find the solution to the set (1, 2) for the boundary conditions

$$\vartheta_1 (x, \ 0) = \vartheta_2 (x, \ 0) = 0; \ \vartheta_2 (0, \ \tau) = \vartheta_0; \ \vartheta_1 (\infty, \ \tau) = \vartheta_2 (\infty, \ \tau) = 0 \tag{3}$$

we transform it to nondimensional form. Since the surface area of the particles per unit volume may be determined as $S = 6(1-\varepsilon)/d_p$ (m^2/m^3), we write

$$\xi = \sqrt{\frac{\alpha^* S}{\lambda_2}} = \sqrt{6(1-\varepsilon)Nu^*}\, \frac{x}{d_p} = A\frac{x}{d_p}\,, \tag{4}$$

$$t = \frac{\alpha^* S}{c_1 \rho_1}\tau = 6(1-\varepsilon)Nu^* Fo = A^2 Fo, \tag{5}$$

$$\theta_1(\xi,\ t) = \frac{\vartheta_1(x,\ \tau)}{\vartheta_0} \text{ and } \theta_2(\xi,\ t) = \frac{\vartheta_2(x,\ \tau)}{\vartheta_0}\,. \tag{6}$$

It follows from (4) and (5) that ξ is a measure of the spacing in kernel diameters ($\xi \sim x/d_p$) and t is proportional to the Fourier number.

In new variables the initial equations (1) and (2) take the form

$$\frac{\partial \theta_1}{\partial t} = \theta_2 - \theta_1, \tag{7}$$

$$\mu\,\frac{\partial \theta_2}{\partial t} = \frac{\partial^2 \theta_2}{\partial \xi^2} - \theta_2 + \theta_1, \tag{8}$$

where $\mu = c_2 \rho_2 / c_1 \rho_1$ is a nondimensional parameter ($\mu \ll 1$). The initial and boundary conditions are written in the following form:

$$\theta_1(\xi,\ 0) = \theta_2(\xi,\ 0) = 0;\ \theta_2(0,\ t) = 1 \text{ and } \theta_1(\infty,\ t) = \theta_2(\infty,\ t) = 0. \tag{9}$$

The thermal flux at the boundary of a surface with constant temperature and disperse medium is

$$q(t) = -\lambda_2\,\frac{\partial \vartheta_2}{\partial x}\bigg|_{x=0} = -\sqrt{\lambda_2 \alpha^* S}\ \vartheta_0\,\frac{\partial \theta_2}{\partial \xi}\bigg|_{\xi=0} \tag{10}$$

and the heat transfer coefficient is correspondingly

$$\alpha(t) = -\sqrt{\lambda_2 \alpha^* S}\,\frac{\partial \theta_2}{\partial \xi}\bigg|_{\xi=0}\,. \tag{11}$$

For convenience in comparison with experimental data, we introduce a nondimensional, time-dependent heat transfer coefficient

$$N(t) = \frac{\alpha(t)}{\sqrt{\lambda_2 \alpha^* S}} = \frac{Nu}{\sqrt{6(1-\varepsilon)Nu^*}} = \frac{Nu}{A} \tag{12}$$

and, consequently,

$$N(t) = -\frac{\partial \theta_2(\xi,\ t)}{\partial \xi}\bigg|_{\xi=0}\,. \tag{13}$$

If we solve the set of differential equations (7) and (8) under conditions (9) and place them in (13), we obtain the desired dependence of Nusselt number on Fourier number. However, using operational techniques, relationship (13) may be found without prior solution of equations (7) and (8).

We introduce the Laplace transforms of the functions θ_1 and θ_2. Then Eqs. (7) and (8) take the form

$$sY_1 = Y_2 - Y_1 \text{ and } \mu s Y_2 = \frac{d^2 Y_2}{d\xi^2} - Y_2 + Y_1 \tag{14}$$

with boundary conditions

$$Y_2(0,\ s) = \frac{1}{s} \text{ and } Y_2(\infty,\ s) = 0. \tag{15}$$

Eliminating Y_1 from Eq. (14), we obtain

$$\frac{d^2 Y_2}{d\xi^2} = \left(1 + \mu s - \frac{1}{1+s}\right)Y_2. \tag{16}$$

Solution of Eq. (16) under conditions (15) has the form

$$Y_2(\xi, s) = \frac{1}{s} \exp\left[-\xi \sqrt{s \frac{1 + \mu + \mu s}{1 + s}}\right] \tag{17}$$

and, consequently,

$$Y_1 = \frac{Y_2}{1 + s}. \tag{18}$$

Using Eq. (17) to obtain the form of the heat transfer coefficient we obtain, finally

$$Q(s) = \frac{1}{\nu s} \sqrt{\frac{1 + \mu + \mu s}{1 + s}}. \tag{19}$$

To find the initial conditions we rewrite Eq. (19) in the form

$$Q(s) = \sqrt{\frac{\mu}{s + 1}} + \sqrt{\frac{\mu}{s + 1}}\left(\sqrt{\frac{s + 2k}{s}} - 1\right), \tag{20}$$

where $(1 + \mu)/\mu$ is denoted by $2k$. Using the integral superposition theorem and performing the elementary transformations, we obtain the original representation for (19)

$$N(t) = \sqrt{\frac{\mu}{\pi}}\left[\frac{\exp(-t)}{\sqrt{t}} + \frac{1 + \mu}{2\mu} J_1\right], \tag{21}$$

where

$$J_1 = \int_0^t (t - \tau)^{-1/2} \exp\left[-t + \tau(1 - k)\right][I_0(k\tau) + I_1(k\tau)]\, d\tau. \tag{22}$$

Using the asymptotic Bessel function expansion in Eq. (22) we obtain a convenient expression for numerical computations for large values of time in the form

$$N(t) = \sqrt{\frac{\mu}{\pi}}\frac{\exp(-t)}{\sqrt{t}} + \sqrt{1 + \mu}\exp\left(-\frac{t}{2}\right)I_0\left(\frac{t}{2}\right). \tag{23}$$

The experimental data shown below in Fig. 1 for comparison are time-averaged values of the heat transfer coefficient. We obtain average values of the nondimensional heat transfer coefficient N(t) by means of integration of the expressions for the instantaneous values in the limits from t_1 to t_2 and then averaging over the given interval:

$$\overline{N}(t) = \sqrt{\frac{\mu}{\pi}}\frac{1}{t_2 - t_1}\int_{t_1}^{t_2}\frac{\exp(-t)}{\sqrt{t}}\, dt + \frac{\sqrt{1 + \mu}}{t_2 - t_1}\int_{t_1}^{t_2}\exp\left(-\frac{t}{2}\right)I_0\left(\frac{t}{2}\right)dt$$

$$= \frac{1}{t_2 - t_1}\sqrt{\mu}\left[\mathrm{erf}(\sqrt{t_2}) - \mathrm{erf}(\sqrt{t_1})\right] + \frac{\sqrt{1 + \mu}}{t_2 - t_1}\left[t_2 {}_1F_1\left(\frac{1}{2}, 2; -t_2\right) - t_1 {}_1F_1\left(\frac{1}{2}, 2; -t_1\right)\right]. \tag{24}$$

Finally, with account of Eqs. (12) and (5)

$$\overline{\mathrm{Nu}} = \frac{A}{\mathrm{Fo}_2 - \mathrm{Fo}_1}\left\{\frac{1}{A^2}\sqrt{\mu}\left[\mathrm{erf}(A\sqrt{\mathrm{Fo}_2}) - \mathrm{erf}(A\sqrt{\mathrm{Fo}_1})\right]\right.$$

$$\left.\sqrt{1 + \mu}\left[\mathrm{Fo}_2 {}_1F_1\left(\frac{1}{2}, 2; -A^2\mathrm{Fo}_2\right) - \mathrm{Fo}_1 {}_1F_1\left(\frac{1}{2}, 2; -A^2\mathrm{Fo}_1\right)\right]\right.. \tag{25}$$

Equation (25) is useful for numerical calculations for $\mathrm{Fo} > 7 \cdot 10^{-3}$ to an error not exceeding 5% for minimal values of Fo.

2. Analysis of Theoretical Relationships Obtained. Consideration of the theoretical relationships obtained shows that the assumed model is in accord with experimental data and is physically correct if a number of compulsory conditions are fulfilled.

1) Discrete test data in the literature pertaining to the time dependence of the heat transfer coefficient for various disperse materials and gases must fall on the general curve in Nu−Fo coordinates with the parameter μ.

2) Theoretical relationships at large values of time must revert to the relationship established from the theory of heat conduction of solid, single-phase media.

Actually, as $t \to \infty$ (practically speaking for $t > 40$), it is possible to write the following expression for Eq. (23) using the asymptotic Bessel representation for Eq. (23)

$$N(t) = \sqrt{\frac{\mu}{\pi}} \frac{\exp(-t)}{\sqrt{t}} + \frac{\sqrt{1+\mu}}{\sqrt{\pi t}},$$

(26)

which takes the following form in the range of t considered

$$N(t) = \frac{1}{\sqrt{\pi t}}.$$

(27)

Transforming to dimensional variables we obtain

$$\alpha = \sqrt{\frac{\lambda_2 (c_1 \rho_1 + c_2 \rho_2)}{\pi \tau}}.$$

(28)

Equation (28) is the known dependence of heat transfer coefficient on time for cooling of a constant temperature plate in an infinite continuous medium, [2]. Along with this the asymptotic representation (26) bounds the region of applicability of the set of differential equations (1) and (2) for disperse media with small values of μ relative to unity.

3) At $t \to 0$ the heat flux must be determined only by the physical properties of the gas phase since the temperature field under this condition is localized within a thin layer of the heat exchange surface and does not "feel" the effects of the disperse phase.

This qualitative conclusion may be confirmed analytically with the aid of the following example. It is known, [3], that, if there is a representation of some function, then its approximate expression for small values of t may be obtained by replacing the exact expression by its asymptotic approximation for large values of s. Expression (19) for $s \gg 1/\mu$ transforms to

$$Q(s) = \sqrt{\frac{\mu}{s}},$$

which corresponds to $N(t) = \sqrt{\mu / \pi t}$ (for $t \ll \mu$). Transforming to dimensional variables we obtain that, for $\tau \ll c_2 \rho_2 / \alpha^* S$

$$\alpha = \frac{q}{\vartheta_0} = \sqrt{\frac{\lambda_2 c_2 \rho_2}{\pi \tau}}.$$

(29)

For the intermediate region ($1 \ll s \ll 1/\mu$) in which $Q(s) = 1/s$

$$N(t) = 1 = \text{const.}$$

(30)

Transforming to dimensional variables we obtain the following expression for intermediate values of time $[(c_2 \rho_2 / \alpha^* S) \ll \tau \ll (c_1 \rho_1 / \alpha^* S)]$

$$\alpha = \frac{q}{\vartheta_0} = \sqrt{\lambda_2 \alpha^* S},$$

(31)

i.e., within a sufficiently large range of variation in nondimensional time, the Nu number must remain independent of time.

We note that, on describing the intensity of the transport process for widely varying combinations of thermophysical and geometric properties of disperse systems, it is necessary to determine the value of a universal constant in addition to the known thermophysical parameters of the components of the disperse system. We shall call this constant the interphase heat transfer constant

$$A = \sqrt{6(1-\varepsilon)Nu^*}.$$

The possibility of experimental determination of the interphase heat transfer constant, which cannot be theoretically determined, permits Eq. (31) to be specified. Actually, reducing Eq. (31) to nondimensional form, we obtain

$$Nu = \sqrt{6(1-\varepsilon)Nu^*} = A.$$

(32)

It follows from this that the interphase heat transfer constant (A) may be found in tests directed toward determination of heat transfer intensity at the surface for small values of Fo, i.e., under conditions of constant Nu.
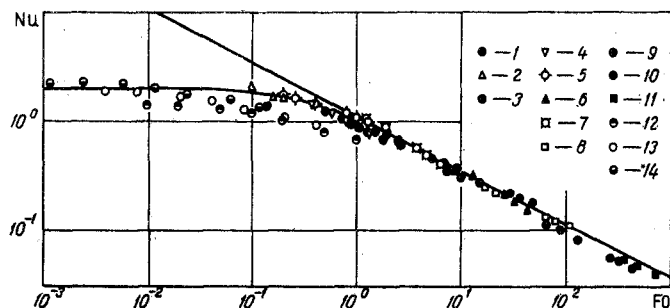
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Fig. 1. Comparison of experimental and theoretical data. Disperse systems: 1, 2) glass beads – air, d_p = 0.38 and 0.475 mm respectively; 3-5) glass beads – Freon-12, d_p = 0.147, 0.200, and 0.38 mm; 6, 7) glass beads – helium, d_p = 0.147 and 0.38 mm; 8) alumina powder – air, d_p = 0.043 mm; 9) alumina powder – Freon-12, d_p = 0.043 mm; 10) alumina powder – helium, d_p = 0.043; 11) mica powder – air, particle size 0.014 mm; 12-14) slag beads – air, d_p = 0.78 mm, 1.2 mm, and 2.2 mm respectively.

We note also that analysis of limiting transitions does not uncover the effects of μ on the value of Nu for any values of Fo. Thus, condition (1) may be improved by formulating it in the following form: all test data must lie on a single curve of Nu = f(Fo) irrespective of the ratio of heat capacity of the phases.

3. Comparison of Theoretical Relationships and Analytical Results with Experimental Data. Before proceeding to quantitative analysis of the theoretical relationships obtained, it is necessary to determine the value of the constant λ_2, the effective heat conductance of the gas phase. We note at the outset that at present, in all probability, it is not possible to obtain the value of λ_2 by a deductive method. In correspondence with the considered relaxation model in this work, it is assumed, to a first approximation, that the effective heat conductance of the gas may be determined as the effective heat conductance of a layer minus the heat conductance of this layer under vacuum conditions (i.e., from particle contacts) which is considered as the effective heat conductance of the solid phase. It should be noted that such a possibility of determining the effective heat conductance of a separate phase from the general effective heat conductance of the layer requires a sound basis.

For comparison with theoretical values, widely known experimental data of Harakas and Beatty, [4], Botterill et al, [5] and Tamarin et al, [6] were taken from the special literature. The experimental data selected for comparison cover a wide range of variation in thermophysical properties of the solid and gas phase, particle dimensions and porosity. The experimental data of Ernst, [7], were not used for comparison of experimental and theoretical values because of the author's error in estimating the value of λ_e as the result of which the values of $\alpha = f(\tau)$ were sharply decreased [8].

The theoretical relationships obtained were generalized in Nu—Fo coordinates for qualitative comparisons of the representations in [4-6] of the time-averaged heat transfer coefficients in annular, mixed layers. Results of computations from Eq. (25) are as follows: for Fo = 0.01, 0.1, 0.16, 0.25, 0.50, 1.0, 2.0, 3.0, 5.0, and 10, $\overline{Nu}$ = 2.0, 1.85, 1.75, 1.62, 1.36, 1.05, 0.77, 0.64, 0.49, and 0.32.

Experimental and theoretical data compared in Fig. 1 show good correspondence between theory and experiment. The observed deviation of some test points from the theoretical curve occurs generally in the range of small values of Fo. This should not be related to the choice of coordinates since, as was shown previously [8], test data fall on one general curve in Nu—Fo coordinates for a properly constituted experiment. It should be noted that the deviation increases with decrease in particle diameter. This fact is found in correspondence with conclusions by the authors in [6] and [9] relating to the fact that, with increase in velocity (for constant values of Fo the relative calorimetric rate in a layer of much finer particles is higher), the possibility of separated flow, which leads to a decrease in heat flux, increases.

As may be seen in Fig. 1, for sufficiently large Fo, the theoretical points fall on a line characteristic for continuous, single-phase media which, to a known degree, serves to confirm the choice of the value of

λ_2. As the Fo number decreases the deviation of theoretical (from the classic Fourier differential heat conduction equation) and test data increases. Here, in a sufficiently broad range of practical interest for values of Fo, the numerical values of Nu number remain constant and nearly equal to 2 ($Nu_{lim} = A = 2$) which also follows in correspondence with the analysis conducted above.

In conclusion it should be noted that choice of an interpolation relationship for the analytical expressions obtained may result in a significant simplification. In first approximation this may be done on the basis of qualitative estimates. The interpolation equation for very small Fo reduces to $N(t) = \sqrt{\mu / \pi t}$, and for large values of Fo to Eq. (27) and is most simply represented in the form

$$N(t) = \frac{\sqrt{\frac{\mu}{\pi t}} + \sqrt{\pi} \, t^n \sqrt{\frac{1+\mu}{\pi t}}}{1 + \pi t^n} . \tag{33}$$

In order that Eq. (33) yield $N(t) = 1$ in the space it is necessary to choose an index of degree $n = 1/2$, i.e.,

$$N(t) = \frac{\sqrt{\mu} + \sqrt{(1+\mu)\pi t}}{\sqrt{\pi t} + \pi t} . \tag{34}$$

Actually, for $\mu \ll t \ll 1$, we have $t \ll \sqrt{t}$, but $\sqrt{(1+\mu)t} \gg \sqrt{\mu}$ and, consequently,

$$N(t) = \frac{\sqrt{\pi} \sqrt{(1+\mu)t}}{\sqrt{\pi} \sqrt{t}} \approx \sqrt{1+\mu} \approx 1, \tag{35}$$

which agrees with Eq. (30) and, since $\mu \approx 1 \cdot 10^{-3}$, then the interval of constant Nu number must be not less than two orders of magnitude; this is also confirmed experimentally. It should be noted, however, that Eq. (34), which was written on the basis of providing an interpolative estimate, is only useful for qualitative illustration. We also note that, with further simplification, Eq. (34) transforms to the expression

$$N(t) = \frac{\sqrt{1+\mu}}{1 + \sqrt{\pi t}} , \tag{36}$$

which is a modification of the Baskakov equation which he presented in a discussion at the Third All-Union Conference on Heat and Mass Transfer (Minsk, 1968) [10].

NOTATION

$c_1\rho_1$	is the volume heat capacity of the solid phase ($c_1\rho_1 = c_s\rho_s(1-\varepsilon)$);
$c_2\rho_2$	is the volume heat capacity of the gas phase ($c_2\rho_2 = c_g\rho_g\varepsilon$);
ϑ	is the temperature;
τ	is the time;
ε	is the porosity;
α, α^*	are the heat transfer coefficient transfer coefficient between the disperse material layer and the surface and the interphase heat transfer coefficient, respectively;
d_p	is the particle diameter;
$Nu = \alpha d_p / \lambda_2$	is the Nusselt number;
$Fo = \lambda_2 \tau / (c_1\rho_1 + c_2\rho_2)d_p^2$	is the Fourier number;
$I_0(x)$	is the modified zero-order Bessel function of the first kind;
$I_1(x)$	is the modified first-order Bessel function of the first kind;
$erf(x)$	is the Cramp function.

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